

Covariance based MIMO Radar Beampattern Design in Non-Uniform Arrays

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Abstract: - Multiple Input Multiple Output (MIMO) radar is an emerging technology which has attracted many researchers in recent years. A MIMO radar system unlike a conventional phased array radar, can choose freely the probing signals transmitted via its antennas to maximize the power around the locations of the targets of interest, or more generally to approximate a given transmit beampattern, and also to minimize the cross correlation of the signals reflected back to the radar by the targets of interest. In this paper, we show how the above desirable features can be achieved by designing the covariance matrix of the probing signal vector transmitted by the radar and location of antennas. Until now many papers have investigated cross correlation matrix only, but in this paper we will include location of transmitter antennas as well to improve the results. And we will show that changing into location of antennas is equivalent to change into carrier frequency. Additionally, we demonstrate the advantages of several MIMO transmit beampattern designs, including a pushing away sidelobes which are next to main lobe design case and to maximize the power around the locations of the targets of interest with producing any extra sidelobes.

Keywords: - MIMO radar, Beampattern matching design, covariance based method, non-uniform arrays.

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) radar is an emerging technology which has drawn considerable attention of many researchers recently. Unlike conventional phased array radars which can only transmit scaled version of a signal, this type of radar can choose freely between transmitter signals which can be correlated or uncorrelated to each other [1], [2].

Generally related to location of antennas, a MIMO radar can be classified into two main categories.

MIMO radar with widely separated antennas[3]

MIMO radar with colocated antennas[4]

In MIMO radars with widely separated antennas, antennas are such far from each other that each of them views different aspects of target, which cause Radar Cross Section (RCS) diversity gain in this type of MIMO radars and can be used to increase the spatial diversity of the system [5], [6], [7]. This RCS diversity can improve the performance of detection [7], angle estimation [8] and Doppler estimation of targets [3] which are discussed and referred [3].

In MIMO radars with colocated antennas, antennas are such close to each other that all of them views identical aspect of a target. Waveform diversity in these types of MIMO radars can be used to increase the spatial resolution [4], excellent interference rejection capability [9], [10], improve parameter identifiability [11], and enhanced flexibility for transmit beampattern design [12], [13] which are discussed and referred [4].

Final goal in colocated MIMO radars which are the main purpose of this paper, is to design suitable waveforms to achieve our desire beampattern, detection, parameter identifiability and etc. Related to waveform design, MIMO radars can be classified into three main categories:

Cross-correlation matrix based design method [12]-[16]

Radar ambiguity function based design method [17]-[20]

Extended targets based design methods [21]-[23]

In the cross-correlation matrix based design methods, which are the main purpose of this paper, the covariance matrix of the waveforms is considered instead of the entire waveform. It is noticeable that a cross-correlation matrix of a collection of waveforms has much less unknown parameters respect to the entire waveforms and it causes easier design. Consequently this kind of design methods affects only the spatial domain. In [12] and [14] the cross-correlation of the transmitted waveforms are designed such that the power can be transmitted to a desired range of angles. In [13] the authors have also designed the cross-correlation matrix of the transmitted waveforms to control the spatial power. However in [13] the cross-correlation between the transmitted signals at a number of given target locations is minimized. This can further increase the spatial resolution in the receiver. In [15] the covariance between the waveforms has been optimized for several design criteria based on the Cramer-Rao bound matrix. In [16] given the optimized cross-correlation matrix, the

corresponding signal waveforms are designed to further achieve low peak-to- average-power ratio (PAR) and higher range resolution.

As it is seen from all the above references, the authors only considered the own cross-correlation matrix. In this paper we wish to consider antenna locations as well and we will show change in the location of antennas is equivalent to change in carrier frequency of waveforms. It will show that with the change in location of antennas, it is possible to improve the capability of MIMO radars. In this paper, with assumption that it is possible to freely choose the location of antennas, we will define two cost functions as follows:
Pushing away the sidelobes

Maximize the power around the target location of interest without producing any extra sidelobes.

The present study has five sections as follows; section I presents a brief introduction. Utilization of cross-correlation matrix and covariance based method formulation for desire problem is discussed in section II. Our main goal and algorithm is provided in section III. Numerical examples and simulation results are discussed in section IV. Section V is focused on conclusion and references are presented at the end of the paper.

II. COVARIANCEBASEDMETHOD

Assume that we have a collection of N transmitter antenna which is located at known coordinates $x_i = (x_{1,i}, x_{2,i}, x_{3,i}) = (x, y, z)$ in some spherical coordinate along the z-axis. In the presented study and in all of the examples and formulas of current paper, it is assume that these transmitter antennas are along the z-axis. It is assume that each transmitter antenna is driven by a specific signal on the carrier frequency of f_c or with wavelength of λ and complex envelope of $s_i(t), i = 1, \dots, N$. At a specific point in space at a distance of r and direction of $k(\theta, \phi)$ from the transmitter antenna, each radiated signal $s_i(t)$ gives rise to a "signal" the far field at radius r , with complex envelope given by

$$y_i(t, r, \theta, \phi) = \frac{1}{\sqrt{4\pi r}} s_i \left(t - \frac{r}{c} \right) e^{j \left(\frac{2\pi}{\lambda} \right) x_i^T k(\theta, \phi)} \quad (1)$$

Where, k is a unit vector in the (θ, ϕ) direction.

At the far field, these signals add linearly and the radiated powers P_i add linearly as well. At this point assume that the i -th element location is on the z-axis at coordinate z_i . The signal at position (r, θ, ϕ) resulting from all of the transmitted signals at far field will be:

$$y(t, r, \theta, \phi) = \sum_{i=1}^N y_i(t, r, \theta, \phi) = \frac{1}{\sqrt{4\pi r}} \sum_{i=1}^N s_i \left(t - \frac{r}{c} \right) e^{j \left(\frac{2\pi z_i}{\lambda} \right) \sin \theta \cos \phi} \quad (2)$$

The power density is of the entire signals then given by

$$P_y(r, \theta, \phi) = \frac{1}{4\pi r^2} \sum_{k=1}^N \sum_{l=1}^N \langle s_k(t) s_l^*(t) \rangle e^{j \left(\frac{2\pi(z_k - z_l)}{\lambda} \right) \sin \theta} \quad (3)$$

And it is known that the complex signal cross-correlation is defined by

$$R_{kl} = \langle s_k(t) s_l^*(t) \rangle \quad (4)$$

With defining the steering vector as below

$$a(\theta) = \left[e^{j \left(\frac{2\pi z_1}{\lambda} \right) \sin \theta}, \dots, e^{j \left(\frac{2\pi z_N}{\lambda} \right) \sin \theta} \right]^T \quad (5)$$

The normalized power density $P(\theta, \phi)$ of signals, in (W/ster), is

$$P(\theta, \phi) = \frac{1}{4\pi} \sum_{k=1}^N \sum_{l=1}^N R_{kl} e^{j \frac{2\pi}{\lambda} (z_k - z_l) \sin \theta} \quad (6)$$

Recognizing that (6) is quadratic form in the Hermitian matrix R which is the cross-correlation matrix of signals, this can be written compactly as

$$P(\theta, \phi) = \frac{1}{4\pi} a^*(\theta) R a(\theta) \quad (7)$$

This normalized power density $P(\theta, \phi)$ is exactly the beampattern which we wish to find[3].

As it is said earlier in this paper, one of the parameters that we want use to optimize the beampattern is location of antennas. It is known that in real world, it may be possible to make an array of antennas with arbitrary antenna locations for a fix beampattern, but for an adaptive beamforming usage it is impossible to have a array with adaptive antenna locations. Then as it is seen from (5) we can change λ instead of z_i and it is mean that for this reason in adaptive usages, we have to use adaptive carrier frequency instead of adaptive antenna locations. It is noticeable that since here from now on, in this paper we will say "change in the location of antennas" only and where it is impossible, it means "change in carrier frequency".

In general case the elements of the signal cross-correlation matrix are complex values except the diagonal elements that they are real. This general case is related to MIMO radars but in the case of phased array radar, all the transmitter signals are correlated with each other and then all the elements in R , are equal to 1.

III. BEAMPATTERNSYNTHESIS

In this section we wish to define our cost functions and introduce our answer to solve them. As it stated before, in this paper we will work with two cost functions:

Pushing away the sidelobes

Maximize the power around the target location of interest without producing any extra sidelobes. In this section we will formulate these cost functions and explain them, and in the next section with some numerical examples we will show how these methods work. It is noticeable that some cost functions like those we wish to consider here, has considered previously on some papers like [13] but the difference in this paper which none of the papers have considered that, is the extra degree of freedom which has achieved by arbitrary antenna locations. This extra degree of freedom will cause more suitable and accurate results.

A. Pushing the sidelobes away

In this section we suppose that for an initial situation, a beampattern of an arbitrary shape with some sidelobes is available. Our goal is to push the sidelobes away from the main beam by the means of optimizing cross-correlation matrix and optimizing of steering vector (optimizing location of antennas). It is noticeable that in many cases like MIMO communication it is desirable that our beampattern doesn't have any sidelobes or as it is possible, the sidelobes be far from the main lobe because the transmitters in communication have noticeable power, or in MIMO radar cases, especially the first sidelobe can hamper adaptive beamforming. Because of these reasons this cost function is an important one which we will consider.

Suppose that the initial beampattern is in the form bellow:

$$P_{\text{int}}(\theta, \phi) = \frac{1}{4\pi} \mathbf{a}_{\text{int}}^*(\theta) \mathbf{R}_{\text{int}} \mathbf{a}_{\text{int}}(\theta) \quad (8)$$

Where P_{int} is an initial beampattern, \mathbf{a}_{int} and \mathbf{R}_{int} are initial steering vector and initial cross-correlation matrix of transmitted signals correspondingly. Sidelobes locations for this initial beampattern would define as follow:

$$\Theta_k, \quad k = 1, \dots, N_{\text{sl}} \quad (9)$$

Where N_{sl} is the number of sidelobes and Θ_k is defined from solving of following equation:

$$\frac{\partial P_{\text{int}}(\theta, \phi)}{\partial (\theta, \phi)} = 0 \quad (10)$$

It is noticeable that as we assumed, our transmitter antennas are linear and along the z-axis, (10) doesn't depend on ϕ and then it is possible to write the above equation in the form bellow:

$$\left. \frac{\partial P_{\text{int}}(\theta)}{\partial (\theta)} \right|_{\theta=\Theta_k} = 0 \quad (11)$$

Also the locations of main lobes are defined as bellow:

$$\Psi_k, \quad k = 1, \dots, N_{\text{ml}} \quad (12)$$

Where N_{ml} is the number of main lobes in the desired beampattern. Then from these information it is possible to introduce the cost function

$$\begin{aligned} \max_{\mathbf{R}, \mathbf{a}} \quad & d_i, d'_i \quad i = 1 - N_{\text{ml}} \\ P(\theta) = & \frac{1}{4\pi} \mathbf{a}^*(\theta) \mathbf{R} \mathbf{a}(\theta) \\ d_i = & |\Theta_i - \Psi_i| \\ d'_i = & |\Theta'_i - \Psi_i| \end{aligned} \quad (13)$$

Where, Θ_i denotes sidelobe in the left of Ψ_i and Θ'_i denotes sidelobe in the right of Ψ_i .

For simplicity and without loss of generality it is assume that the desired beampattern and the initial beampattern have both one main beam which is centered at the boresight at $\theta = 0$ angle and the goal is to push the side lobes away from this main lobe. with this assumption, one can write the (11) expression in the following form:

$$\begin{aligned} \left. \frac{\partial P_{\text{int}}(\theta)}{\partial(\theta)} \right|_{\theta=\Theta_k} &= 0 \quad \Theta_k > 0 \\ \Theta'_k &= -\Theta_k \quad k = 1 \sim \frac{N_{\text{sl}}}{2} \end{aligned} \tag{14}$$

And then main lobe can be written as bellow:

$$\Psi = \theta_k |_{P(\theta_k)=\max P} \tag{15}$$

So Θ and Θ' would be the locations of nearest sidelobes to the main lobe form right and left corespondingly. With these simplifier assumptions the (13) cost function could be written in the simpler following format:

$$\begin{aligned} \max_{R, a} \quad & d, d' \\ P(\theta) &= \frac{1}{4\pi} a^*(\theta)Ra(\theta) \\ d &= |\Theta_i - \Psi| \\ d' &= |\Theta'_i - \Psi| \end{aligned} \tag{16}$$

As it is seen, the above equation is an optimization problem respect to two variables: cross-correlation matrix of signals, R , and steering vector which is related to location of antennas placement. Then it is a two variable optimization problem which is difficult to solve directly. Because of that, here we will introduce an iterative algorithm to solve this problem step by step. Actually in this algorithm we will optimize this equation for each of the variables separately and put the result of each step of iteration in other equation. Now we wish introduce the algorithm.

The steps of this algorithm would be as follow:

1. First take initial values both for cross-correlation matrix of transmitted signals, R , and for steering vector of a .
2. Obtain transmitter beampattern by means of latter step and (7) equation
3. Finding the first two closest sidelobes to the main lobe by means of step number 2 and finding

$$\begin{aligned} d &= |\Theta_i - \Psi_i| \\ d' &= |\Theta'_i - \Psi_i| \end{aligned}$$

Distances of these sidelobes from the main lobe.

4. Finding an optimize cross-correlation matrix of transmitted signals from bellow equation:

$$\begin{aligned} \max_R \quad & d, d' \\ P(\theta) &= \frac{1}{4\pi} a^*(\theta)Ra(\theta) \\ d &= |\Theta_i - \Psi_i| \end{aligned}$$

$$d' = |\Theta'_i - \Psi_i|$$

5. Set the initial cross-correlation matrix to R which is obtained from the last step
6. Obtain transmitter beampattern by means of latter step and (7) equation
7. Finding the first two closest sidelobes to the main lobe by means of step number 2 and finding

$$d = |\Theta_i - \Psi_i|$$

$$d' = |\Theta'_i - \Psi_i|$$

Distances of these sidelobes from the main lobe.

8. Finding an optimize cross-correlation matrix of transmitted signals from bellow equation for constant value for steering vector for constant values for cross-correlation matrix:

$$\max_a \quad d, d'$$

$$P(\theta) = \frac{1}{4\pi} a^*(\theta) R a(\theta)$$

$$d = |\Theta_i - \Psi_i|$$

$$d' = |\Theta'_i - \Psi_i|$$

9. Set the initial steering vector to a which is obtained from the last step
10. Repeat steps from step number 2

It is possible to repeat these steps to obtain a desire distance between main lobe and sidelobes. In the section of "numerical examples" it will be shown that how this algorithm works.

B. Maximize the power around the target locations of interest without producing any extra sidelobes

In [13] the authors have introduced a cost function which it is purpose is to maximize the transmitted power around some specific target locations in the form bellow:

$$\begin{aligned} \max_R \quad & \text{tr}(R\hat{B}) \quad \text{subject to} \\ & \text{tr}(R) = \text{const} \\ & R \geq 0 \end{aligned} \tag{17}$$

Where \hat{B} is an estimation matrix of B which include targets locations information. This matrix is represented by:

$$B = \sum_{k=1}^K a(\theta_k) a^*(\theta_k) \tag{18}$$

But in this paper and in this section it is assumed that the main goal is to maximize the power transmitted around the target locations without producing any extra sidelobes. Moreover, in this paper it is assumed that one degree of freedom is also available, which is beside the cross-correlation matrix of transmitter signals, location of antenna placements is has to be designed.

It is noticeable that in many radar cases like target tracking, it is desirable to maximize the transmitted power around some specific locations. In conventional phased array radars the number of targets which can simultaneously being tracked is fewer than MIMO radars furthermore, the algorithm that is introduce in (17) by authors for this goal, is not a suitable algorithm because this algorithm may produce some extra sidelobes and different concentrating to different targets in the case with a set of target is not possible with that algorithm. Because of these reasons, this cost function has been introduced in this paper to maximize the power around the targets and not to produce any extra sidelobes as well. More ever as it is known and some papers have shown so far, it is possible to place antenna elements in order to increase the gain in some directions but all of that algorithms produce extra sidelobes in the final beampattern, because of that to prevent producing extra sidelobes

this constraint has been added here. In other words this cost function is equivalent to make the main lobes sharper and to reduce the sidelobe level.

Just like the previous section, it is assumed that there is an initial beampattern here too, with some main lobes and sidelobes. Main lobes are defined as

$$\Psi_k, \quad k = 1, \dots, N_{ml} \tag{19}$$

And sidelobes are defined as

$$\Theta_k, \quad k = 1, \dots, N_{sl} \tag{20}$$

Where in these equations N_{ml} is the number of main lobes and N_{sl} is the number of sidelobes in the beampattern correspondingly. Then the general cost function will be the form bellow

$$\begin{aligned} \max_{R, a} \quad & \text{tr}(R\hat{B}) \quad \text{subject to} \\ |a_{1,i}| = 1 \quad & i = 1, \dots, N_{\text{antenna}} \\ N_{sl} \leq \text{const} \\ \text{tr}(R) = \text{const} \\ R \geq 0 \end{aligned} \tag{21}$$

Where \hat{B} is an estimation matrix of B which include targets locations information and $|v|$ denotes absolute value of v . This matrix is represented by:

$$\begin{aligned} B &= \sum_{k=1}^K a(\theta_k) a^*(\theta_k) \rightarrow \\ \hat{B} &= \sum_{k=1}^K a(\theta'_k) a^*(\theta'_k) \end{aligned} \tag{22}$$

Where in this optimization problem, $|a| = 1$ denotes that each element of steering vector has unity absolute, $N_{sl} \leq \text{const}$ denotes that number of sidelobes doesn't exceed it means that it can reduce or being constant but it cant increase, $\text{tr}(R) = \text{const}$ denotes that the total transmitted power is constant, $R \geq 0$ denotes that cross-correlation matrix of transmitted signals is a nonnegative definite matrix, and θ'_k denotes an estimation of k -th target location which by means of that, \hat{B} will be produced.

Like the previous section it is seen that it is hard to solve this two variable optimization problem analytically, then here in this paper we are going to introduce an iterative step by step algorithm to solve this problem. Actually in this iterative algorithm, we wish to optimize the problem for each of the variables separately and then put the result of each step of iteration in the other problem. The step by step iterative algorithm is as follow:

1. First take initial values both for cross-correlation matrix of transmitted signals, R , and for steering vector of a .
2. Obtain transmitter beampattern by means of latter step and (7) equation
3. Finding N_{sl} the number of sidelobes of the beampattern
4. As it is assumed that there is estimation for targets locations and taking constant values for steering vector, the following matrix would be compute.

$$\hat{B} = \sum_{k=1}^K a(\theta'_k) a^*(\theta'_k)$$

5. Finding an optimize cross-correlation matrix of transmitted signals from bellow equation for constant value for steering vector:

$$\max_R \quad \text{tr}(R\hat{B}) \quad \text{subject to}$$

$$\begin{aligned} N_{sl} &\leq \text{const} \\ \text{tr}(\mathbf{R}) &= \text{const} \\ \mathbf{R} &\geq 0 \end{aligned}$$

6. Set the initial cross-correlation matrix to \mathbf{R} which is obtained from the last step
7. Obtain transmitter beampattern by means of latter step and (7) equation
8. Finding N_{sl} the number of sidelobes of the beampattern
9. Finding an optimize cross-correlation matrix of transmitted signals from bellow equation for constant values for cross-correlation matrix:

$$\begin{aligned} \max_{\mathbf{a}} \quad & \text{tr}(\mathbf{R}\hat{\mathbf{B}}) \quad \text{subject to} \\ & N_{sl} \leq \text{const} \\ & |\mathbf{a}| = 1 \end{aligned}$$

10. Set the initial steering vector to \mathbf{a} which is obtained from the last step
11. Repeat steps from step number 2

It is possible to repeat these steps to obtain a desire distance between main lobe and sidelobes. In the section of "numerical examples" it will be shown that how this algorithm works. It is noticeable that the introduced algorithm is one way to solve the two variables optimization problem, surely there are ways which solve that problem faster or more optimum that has been left as future works.

IV. NUMERICALEXAMPLES

In this section we are going to examine the performance of the algorithms which has been introduced in the previous sections, with some real and numerical examples. It is noticeable that in this section and in all the examples, it is assumed that there is an array with 7 transmitter antenna elements, which are placed along the z-axis. The result beampattern is measured at distance of 1000 km from the x-axis and in ranges of -1000km to 1000km along the z-axis.

A. Pushing the sidelobes away

In this section it is desirable to consider the performance of "Pushing the sidelobes away" algorithm, and with multiple figures, the procedure of the algorithm will be shown.

First some initial values should be taken. In this example a 7 element uniform phased array with wave length spacing between elements is taken for initial beampattern which the goal is to push it's sidelobes away from the main lobe. Fig. 1 shows the result beampattern after doing step number 4 from the steps of the algorithm, compare to initial beampattern.

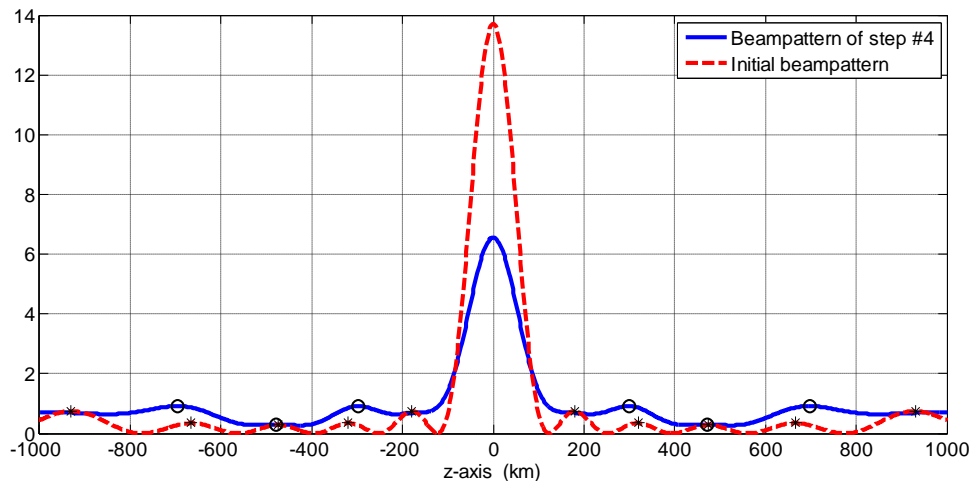


Fig. 1.comparison between initial beampattern and beampattern of step #4

In this figure red plot is related to a phased array radar with 7 transmitter antennas and of wave length spacing, blue plot is related to beampattern which is resulted from step number 4 form the iterative algorithm, "o"

denotes the locations of sidelobes in computed beampattern and "*" denotes the locations of sidelobes in initial beampattern.

As it is seen from this figure after this step the location of first sidelobe level has migrate to a point farther than the initial first sidelobe location in initial beampattern. After this step, the computed cross-correlation matrix is fixed and the optimization problem will be solved due to antenna placement. Fig. 2 shows the result of this algorithm respect to latter step. It is noticeable that in each step of optimization algorithm, the first point which satisfied the problem is taken and algorithm will go to next step.

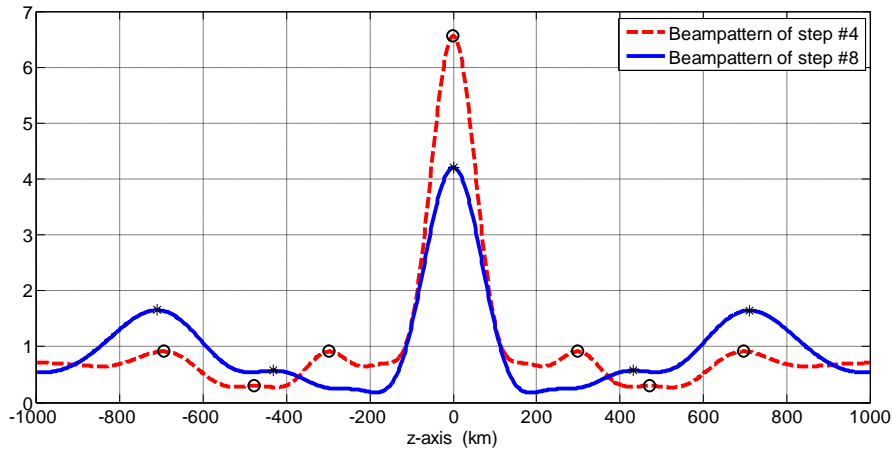


Fig. 2.comparison between beampattern of step #4 and beampattern of step #8

In this figure, the red plot is related to latter beampattern wish has achieved in step number 4 and the blue plot is related to beampattern which is gained from step number 8 in the algorithm for fixed value of cross-correlation matrix and variable steering vector, "o" and "*" are sidelobes for beampattern of #4 and #8 steps respectively. As it is seen from this figure with optimizing the locations of transmitter antennas it is possible to push the sidelobes away. Now one can repeat this algorithm more and more times to get his desire beampattern and resolution. Fig. 3 shows the beampattern resulted from 5 time iteration of this algorithm and the result is compared to the initial beampattern of phased array.

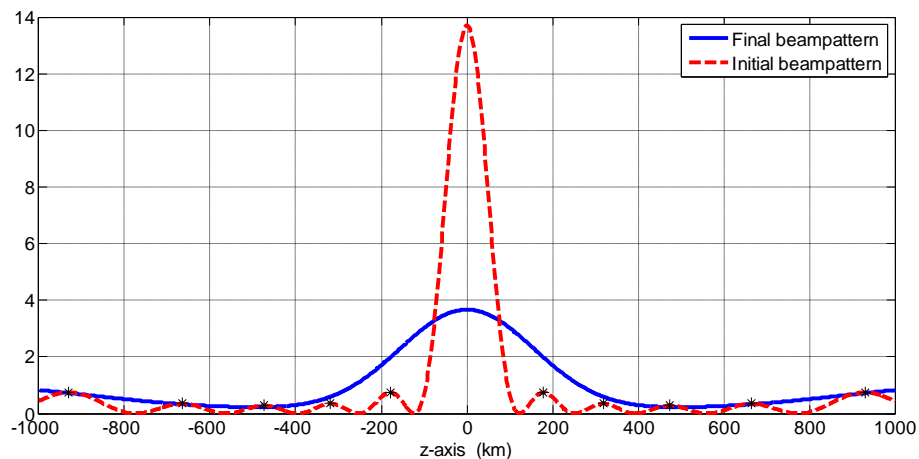


Fig. 3.comparison between initial beampattern and final beampattern after 5 times iteration

In this figure as it is seen, the red plot is the initial beampattern and the blue one is final beampattern. It is seen that after five iterations there will not be any sidelobes in final beampattern although in this case gain and directivity of transmitted beam and system is decreased, this a desirable beampattern for some applications. To preventing of much loss in gain and directivity it is possible to add one more constraint to the optimization problem like as:

$$\max P_{\text{final}} > \frac{1}{2} \max P_{\text{int}} \quad (23)$$

Where, P_{final} denotes final beampattern and P_{int} denotes initial beampattern. Fig. 4 shows the beampattern resulted from 5 time iteration of the algorithm including the extra constraint in (23), and the result is compared to the initial beampattern of phased array.

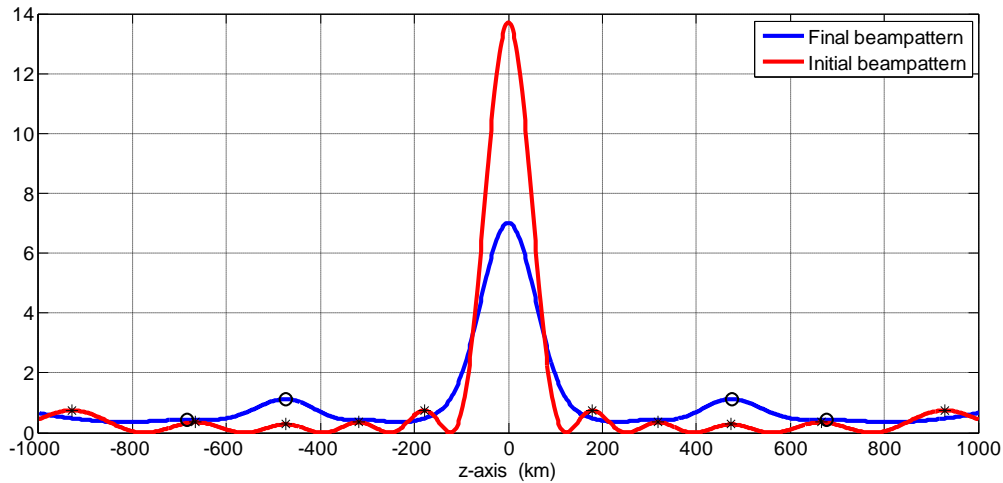


Fig. 4.comparison between initial beampattern and final beampattern after 5 times iteration with extra constraint of (23)

In this figure as it is seen, the red plot is the initial beampattern and the blue one is final beampattern. It is seen that after five iterations the first sidelobe of final beampattern will be farther than third sidelobe of initial beampattern further more in this case gain of final beampattern is greater than half of initial beampattern.

As a result it is seen that with this cost function and the introduced algorithm, it is possible to take the sidelobes away from the main lobes.

B. Maximize the power around the target locations of interest without producing any extra sidelobes

In this section it is desirable to consider the performance of "Maximize the power around the target locations of interest without producing any extra sidelobes" algorithm, and with multiple figures, the procedure of the algorithm will be shown.

First some initial values should be taken. In this example a 7 element uniform phased array with half wave length spacing between elements is taken for initial beampattern which the goal is to decrease sidelobe levels and increase the gain and directivity of the initial beampattern. Fig. 5 shows the result beampattern after doing step number 5 from the steps of the algorithm, compare to initial beampattern. It is noticeable that as in this figure the initial beampattern doesn't have any sidelobes, beampattern of each steps must not have any sidelobes too.

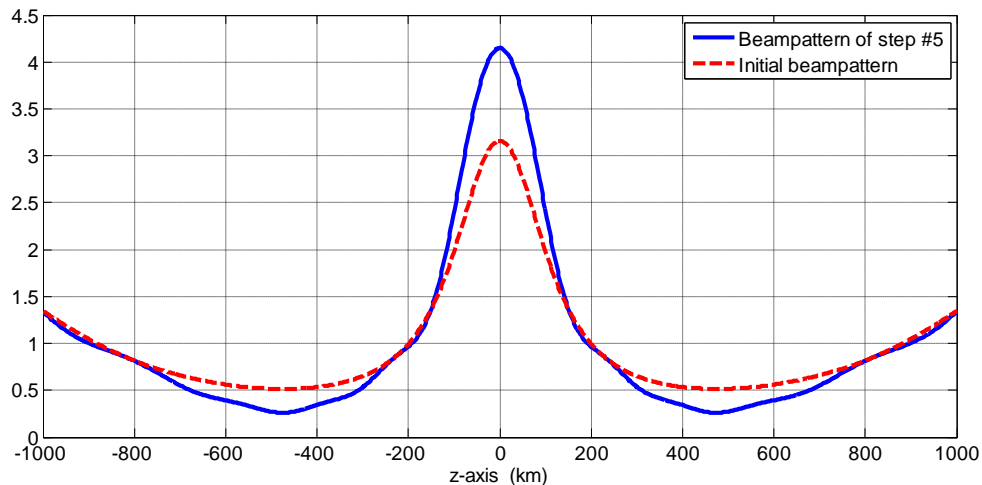


Fig. 5.comparison between initial beampattern and beampattern of step #5

In this figure red plot is related to a phased array radar with 7 transmitter antennas and of half wave length spacing, blue plot is related to beampattern which is resulted from step number 5 form the iterative algorithm.

As it is seen from this figure after this step the resulting beampattern doesn't have sidelobes yet. After this step, the computed cross-correlation matrix is fixed and the optimization problem will be solved due to antenna placement. Fig. 6 shows the result of this algorithm respect to latter step. It is noticeable that in each step of optimization algorithm, the first point which satisfied the problem is taken and algorithm will go to next step.

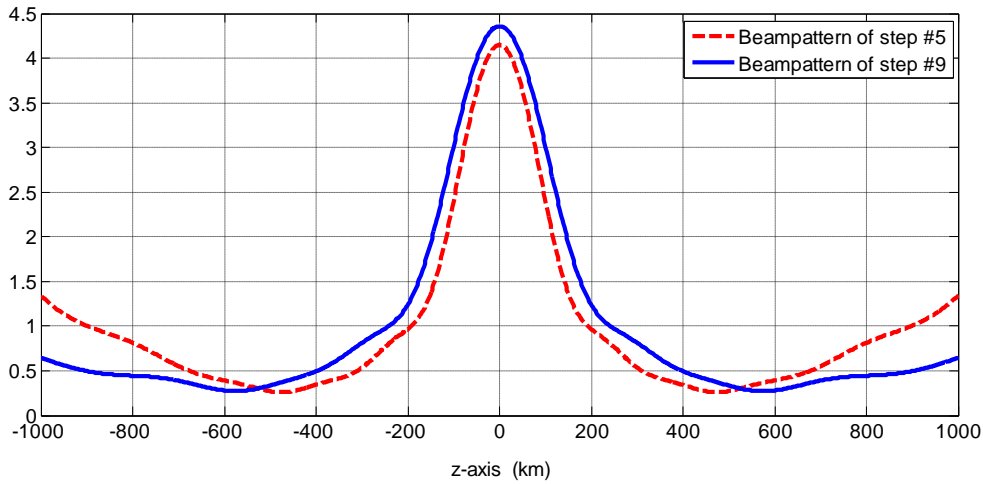


Fig. 6. comparison between beampattern of step #5 and beampattern of step #9

In this figure, the red plot is related to latter beampattern which has achieved in step number 5 and the blue plot is related to beampattern which is gained from step number 9 in the algorithm for fixed value of cross-correlation matrix and variable steering vector. As it is seen from this figure with optimizing the locations of transmitter antennas it is possible to maximize power around desired locations of interest. Now one can repeat this algorithm more and more times to get his desired beampattern and resolution. Fig. 7 shows the beampattern resulted from 5 time iteration of this algorithm and the result is compared to the initial beampattern of phased array.

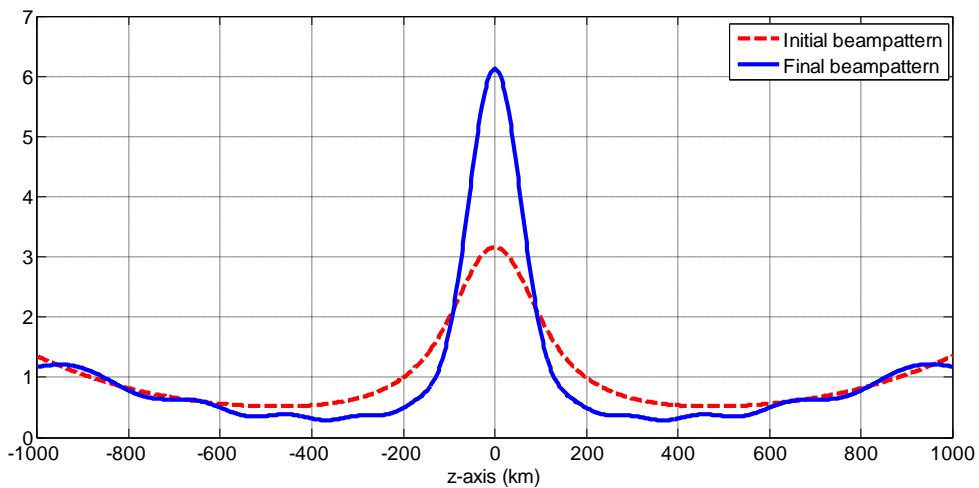


Fig. 7. comparison between initial beampattern and final beampattern after 5 times iteration of algorithm

In this figure the red plot is the initial beampattern and the blue one is final beampattern which is resulted from five time iteration of the introduced algorithm. It is noticeable that one can continue the algorithm for desired result and resolution. As it is seen from this figure with the introduced algorithm it is possible to maximize transmitted power around some targets of interest without producing any extra sidelobes. In other words, as it is shown in Fig. 7 it is possible to reduce total sidelobe level and increase the gain and directivity of transmitted beampattern.

Special case of phased array

In this subsection it is desirable to implement the algorithm for the special case of phased array radar. As it is known for a phased array radar all the transmitted signals are correlated with each other then, there is no degree

of freedom in choosing cross-correlation matrix of R , so in the introduced algorithm one can just implement steps which are related to designing of steering vector of a . Fig. 8 shows the result of such algorithm for a 7 elements antenna array. It is noticeable that in this figure the initial beampattern is related to 7 elements antenna array with half wave length spacing between its elements.

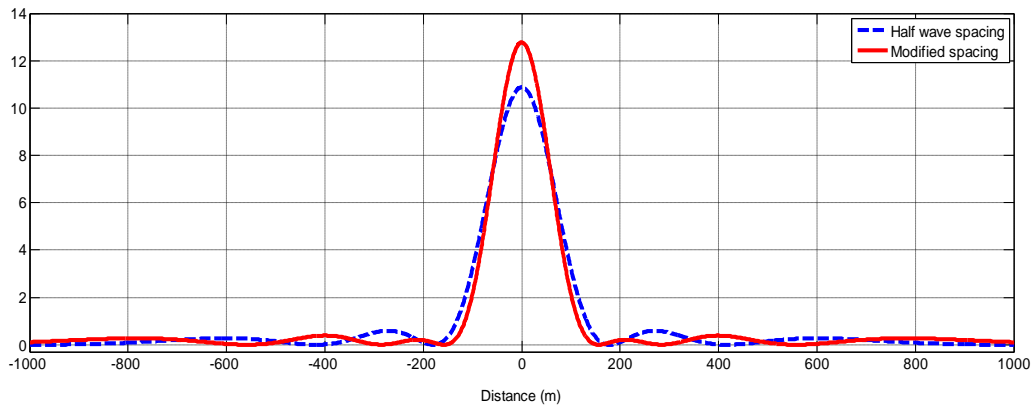


Fig. 8.result of introduced algorithm for special case of phased array

As it is seen from this figure, this algorithm can be used for phased array radar too.

V. CONCLUSION AND FUTURE WORKS

We have considered several transmit beampattern design problems for MIMO radar systems. We have shown that through beampattern design, by focusing the transmit power around the locations of the targets of interest while pushing the sidelobes away or increasing system gain, we can significantly improve the parameter estimation accuracy. We have also shown that with change in steering vector or change in to locations of transmitter antennas it is possible to improve the results. It is noticeable that the two algorithm. which has been introduced in this paper can be complementary to each other. It means that we can first push the sidelobes away from the main lobes respect to first algorithm, and then by means of second algorithm increase the gain and directivity of antenna transmitted beampattern. As it presented in this paper, we solve the two variable optimization problem with some suboptimal solution, as a future work one can solve that problem directly.

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